Key

Final Exam Review **Common Core Math 2**

Factor each expression completely

1. $p^2 - 12p + 27$
$(p-9)(p-3)$

2. $w^2 - w - 6$
$(w-3)(w+2)$

3. $x^2 - 196$
$(x-14)(x+14)$

4. $2x^2 + 12x + 18$
$2(x^2+6x+9)$
$2(x+3)(x+3)$

5. $25x^2 - 49y^2$
$(5x-7y)(5x+7y)$

6. $m^3 + 13m^2 + 42m$
$m(m^2+13m+42)$
$m(m+7)(m+6)$

7. $5x^2 + 20x - 105$
$5(x^2+4x-21)$
$5(x+7)(x-3)$

8. $81x^4 - y^4$
$(9x^2-y^2)(9x^2+y^2)$
$(3x-y)(3x+y)(9x^2+y^2)$

Solve each quadratic equation using the method indicated:

9. Factoring: $5t^2 - 10t - 120 = 0$
$5(t^2-2t-24)$
$5(t-6)(t+4)$
$t=6,\ t=-4$

10. Factoring: $m^2 + 6m - 23 = -7$
$m^2+6m-16=0$
$(m+8)(m-2)$
$m=-8,\ m=2$

11. Factoring: $r^2 - 35 = -2r$
$r^2+2r-35=0$
$(r+7)(r-5)$
$r=-7,\ r=5$

12. Quadratic Formula: $5d^2 - 6d - 20 = 0$
$\frac{6\pm\sqrt{6^2-4(5)(-20)}}{2(5)}$
$\frac{6\pm\sqrt{436}}{10}$
$\frac{6\pm 2\sqrt{109}}{10} = \frac{3\pm\sqrt{109}}{5}$

13. Quadratic Formula: $3x^2 - 24x = 4x$
$3x^2-28x=0$
$\frac{28\pm\sqrt{28^2-4(3)(0)}}{2(3)}$
$\frac{28\pm\sqrt{784}}{6}$
$=\frac{28\pm 28}{6} = 0, \frac{28}{3}$

14. Quadratic Formula: $5n^2 + 8n - 10 = 0$
$\frac{-8\pm\sqrt{8^2-4(5)(-10)}}{2(5)}$
$\frac{-8\pm\sqrt{264}}{10}$
$\frac{-8\pm 2\sqrt{66}}{10}$
$\frac{-4\pm\sqrt{66}}{5}$

15. Square Root: $3(x+5)^2 = 54$
$(x+5)^2 = 18$
$x+5 = \pm\sqrt{18}$
$x+5 = \pm 3\sqrt{2}$
$x = -5\pm 3\sqrt{2}$

16. Square Root: $25x^2 + 7 = 52$
$25x^2 = 45$
$x^2 = \frac{9}{5}$
$x = \pm\frac{3}{\sqrt{5}}$
$x = \pm\frac{3\sqrt{5}}{5}$

Graphing:

17. $y = x^2 + 8x + 12$

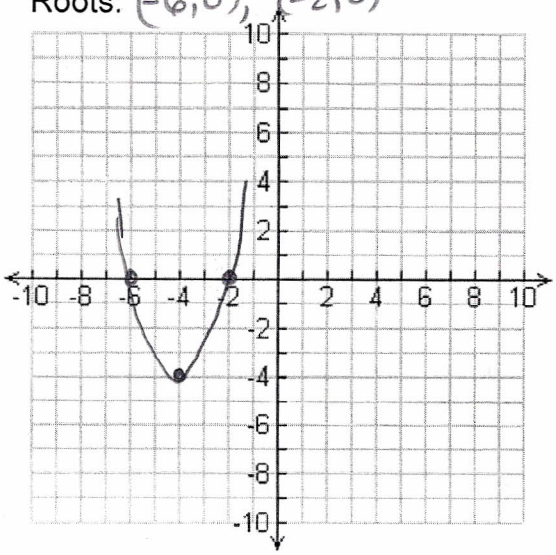
Axis of Symmetry: $x = \frac{-8}{2(1)} = -4$

Vertex: $(-4, -4)$

y-intercept: $(0, 12)$

$(x+6)(x+2)$
$x=-6 \quad x=-2$

Roots: $(-6, 0)$, $(-2, 0)$



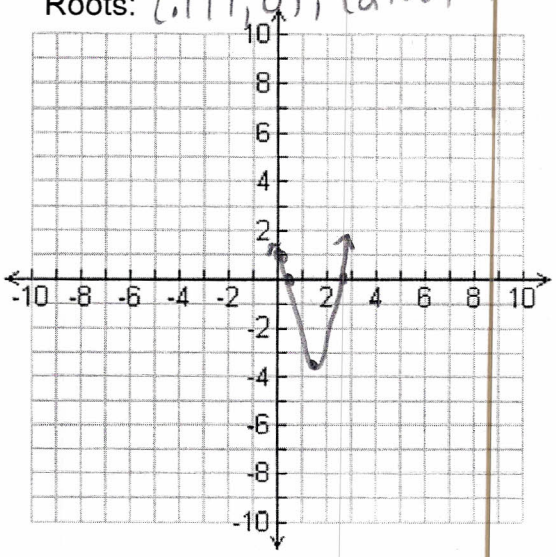
18. $y = 2x^2 - 6x + 1$

Axis of Symmetry: $x = \frac{6}{2(2)} = \frac{6}{4} = \frac{3}{2}$

Vertex: $(\frac{3}{2}, -3.5)$

y-intercept: $(0, 1)$

Roots: $(.177, 0)$, $(2.82, 0)$



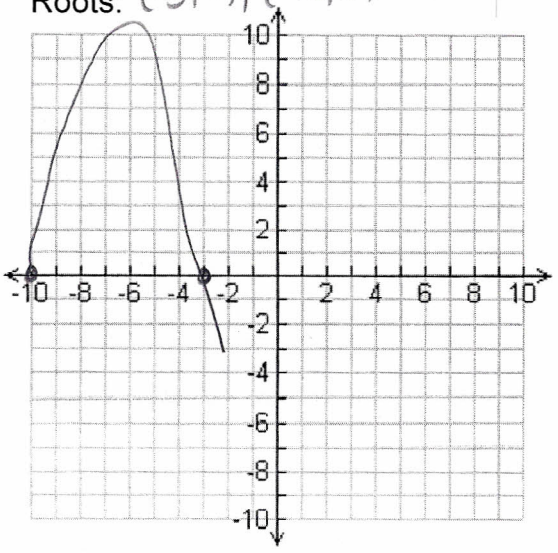
19. $y = -2x^2 - 26x - 60$

Axis of Symmetry: $x = -6.5$

Vertex: $(-6.5, 24.5)$

y-intercept: $(0, -60)$

Roots: $(-3, 0)$, $(-10, 0)$



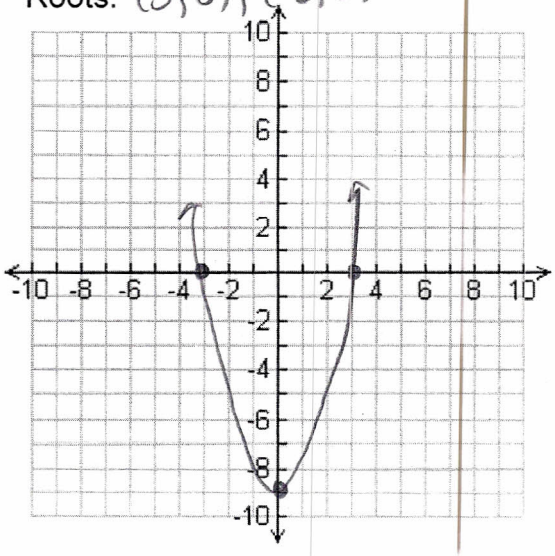
20. $y = x^2 - 9$

Axis of Symmetry: $x = 0$

Vertex: $(0, -9)$

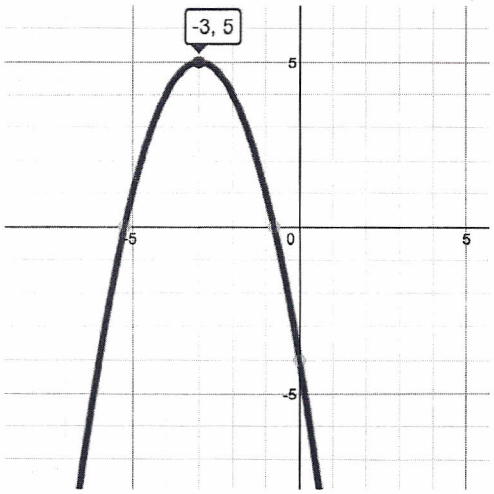
y-intercept: $(0, -9)$

Roots: $(3, 0)$, $(-3, 0)$



State the increasing/decreasing intervals for each:

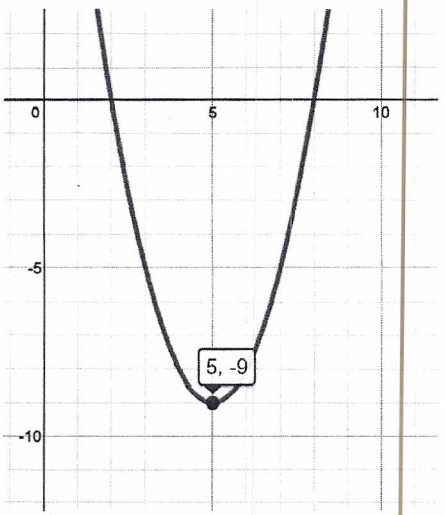
21.



Increasing: $x < -3$

Decreasing: $x > -3$

22.



Increasing: $x > 5$

Decreasing: $x < -5$

Describe each Transformation from the parent graph: $y = x^2$

24. $y = (x+7)^2 - 3$ left 7, down 3

25. $y = -3(x-18)^2$ reflect, stretch, right 18

26. $y = -x^2 + 12$ reflect, up 12

27. $y = \frac{3}{8}(x-5)^2 - 21$ shrink, right 5, down 21

28. $y = \frac{11}{4}x^2$ stretch

29. $y = 2x^2 - 16x + 37$ (put in vertex form first)

$y = 2(x-4)^2 + 5$ stretch, right 4, up 5

Solve each system of equations.

30. $x^2 - 10x - 3y - 16 = 0$
$x + y + 4 = 0$

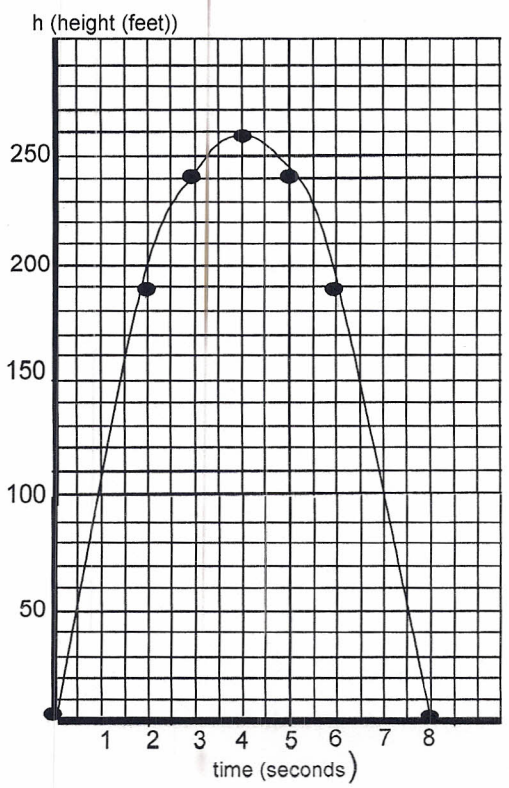
$-x^2 - 10x - 3(-x-4) - 16 = 0$
$-x^2 - 10x + 3x + 12 - 16 = 0$
$-x^2 - 7x - 4 = 0$
$x^2 + 7x + 4 = 0$
$y = -x - 4$
$(-6.4, 2.4)$
$(-.63, -3.37)$

31. $3x^2 + 3y^2 + 5x + 3y + 2 = 0$
$x - 3y = -1$

$x = 3y - 1$ $(-1, 0)$

Applications of Quadratics

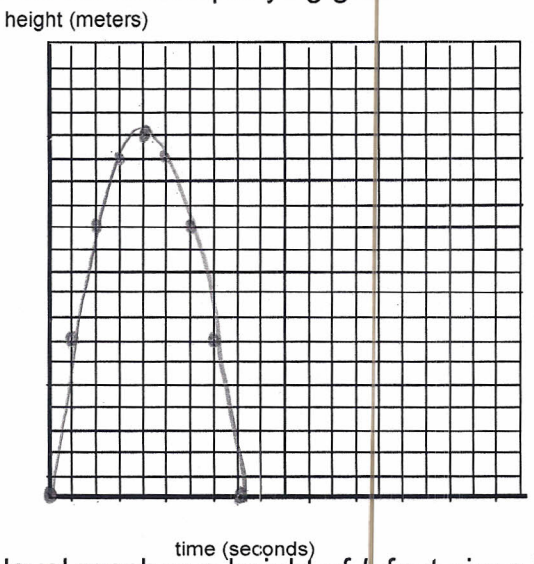
32. Using the graph at the right, it shows the height **h** in feet of a small rocket **t** seconds after it is launched. The path of the rocket is given by the equation: $h = -16t^2 + 128t$.



a. How long is the rocket in the air? 8 sec
b. What is the greatest height the rocket reaches? 260 ft
c. About how high is the rocket after 1 second? 506 ft
d. After 2 seconds,
   a. about how high is the rocket? 190
   b. is the rocket going up or down? up
e. After 6 seconds,
   a. about how high is the rocket? 190
   b. is the rocket going up or down? down
f. Using the equation, find the **exact** value of the height of the rocket at 2 seconds.

$h = -16(2)^2 + 128(2)$
$h = 192$ ft

33. A ball is thrown in the air. The path of the ball is represented by the equation $h = -t^2 + 8t$. Graph the equation over the interval $0 \le t \le 8$ on the accompanying grid.



What is the maximum height of the ball? 16m

How long is the ball above 7 meters? 6sec

a) $\frac{-8}{2(-1)} = 4$ $\quad h = -(4)^2 + 8(4) = 16$

b) $7 = -t^2 + 8t$

$t^2 - 8t + 7 = 0$

$(t-7)(t-1)$

7, 1

34. After t seconds, a ball tossed in the air from the ground level reaches a height of h feet given by the equation $h = 144t - 16t^2$.

a. What is the height of the ball after 3 second? $144(3) - 16(3)^2 = 288$

b. What is the maximum height the ball will reach? 324 ft

c. Find the number of seconds the ball is in the air when it reaches a height of 224 feet.

$224 = 144t - 16t^2$ $\quad t^2 - 9t + 14 = 0$ $\quad (t-7)(t+2)$ $\quad t=7 \quad t=-2$ $\quad$ 7sec

$16t^2 - 144t + 224 = 0$

d. After how many seconds will the ball hit the ground before rebound?

$0 = 144t - 16t^2$ $\quad t=0$

$16t(9-t)$ $\quad t=9$

35. A rocket carrying fireworks is launched from a hill 80 feet above a lake. The rocket will fall into lake after exploding at its maximum height. The rocket's height above the surface of the lake is given by $h = -16t^2 + 64t + 80$.

a. What is the height of the rocket after 1.5 second? 140 ft

b. What is the maximum height reached by the rocket? 144 ft

c. How long will it take for the rocket to hit 128 feet? 1 sec

d. After how many seconds after it is launched will the rocket hit the lake? 5

36. A rock is thrown from the top of a tall building. The distance, in feet, between the rock and the ground t seconds after it is thrown is given by $d = -16t^2 - 4t + 382$. How long after the rock is thrown is it 370 feet from the ground?

$370 = -16t^2 - 4t + 382$

$16t^2 + 4t - 12 = 0$

$4t^2 + t - 3$

$t^2 + t - 12$

$(t + \frac{4}{4})(t - \frac{3}{4})$

$(t+1)(4t-3)$

$t=-1 \quad t=3/4$

$t = 3/4$

Solve equations

37. $12 = \sqrt{b-5} + 9$

$3 = \sqrt{b-5}$

$9 = b-5$

$14 = b$

38. $-7\sqrt{n-9} = -7$

$\sqrt{n-9} = 1$

$n-9 = 1$

$n = 10$

39. $\sqrt{2m-12} = \sqrt{m-2}$

$2m-12 = m-2$

$m-12 = -2$

$m = 10$

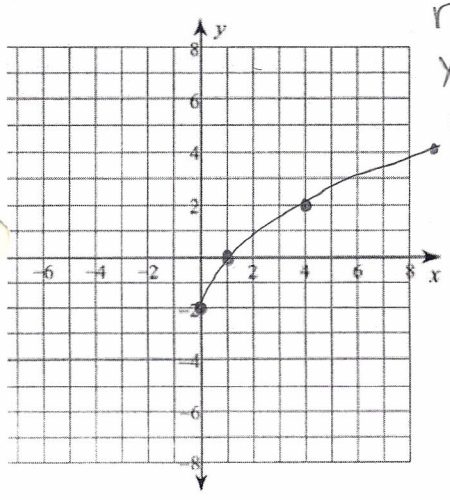
40. $14 = (x+24)^{\frac{1}{2}}$

$196 = x+24$

$172 = x$

Sketch the graph. Identify the domain and range of each. Find the x-intercept and y-intercept. Describe the transformation from $y = \sqrt{x}$

41. $y = -2 + 2\sqrt{x}$



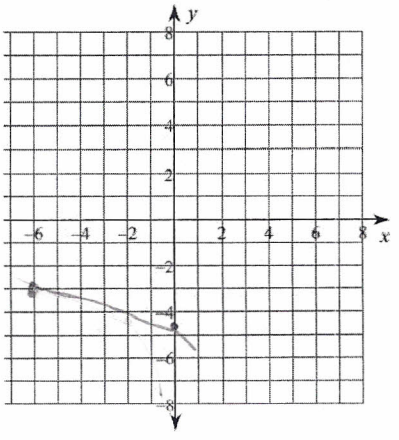
d: $x \geq 0$

r: $y \geq -2$

x-int: (1,0)

y-int: (0,-2)

42. $y = -\frac{3}{4}\sqrt{x+6} - 3$



d: $x \geq -6$

r: $y \leq -3$

x-int: none

y-int: (0,-4.8)

Write each expression in exponential form.

43. $(\sqrt[3]{4x})^5$ $(4x)^{5/3}$

44. $\sqrt[5]{v^3}$ $v^{3/5}$

Write each expression in radical form.

45. $(7m)^{\frac{1}{2}}$ $\sqrt{7m}$

46. $(6n)^{\frac{4}{3}}$ $(\sqrt[3]{6n})^4$

Simplify each expression.

48. $(81x^2)^{\frac{3}{2}}$ $729x^3$

49. $(a^9)^{\frac{1}{3}}$ a^3

50. $(125m^3)^{\frac{4}{3}}$ $625m^4$

51. $(64n^9)^{\frac{1}{3}}$ $4n^3$

Write the equation for the radical function with the described transformation.

52. vertical shrink of 2/5, shifted right 3, shifted up 6 and reflected across the x-axis. $y = -\frac{2}{5}\sqrt{x-3} + 6$

53. vertical stretch of 7, shifted down 4 $y = 7\sqrt{x} - 4$

Write an equation after the transformation from the parent graph of $y = \sqrt[3]{x}$ and $y = x^3$.

54. Vertical stretch by a factor of 4, right 5, reflection over the x-axis $y = -4\sqrt[3]{x-5}$, $y = -4(x-5)^3$

55. left 4, down 8, vertical shrink by a scale factor of ¾ $y = \frac{3}{4}\sqrt[3]{x+4} - 8$, $y = \frac{3}{4}(x+4)^3 - 8$

Describe the transformation.

56. Parent graph: $y = x^3$

$y = \frac{10}{3}(x+7)^3 - 12$

stretch $\frac{10}{3}$, left 7, down 12

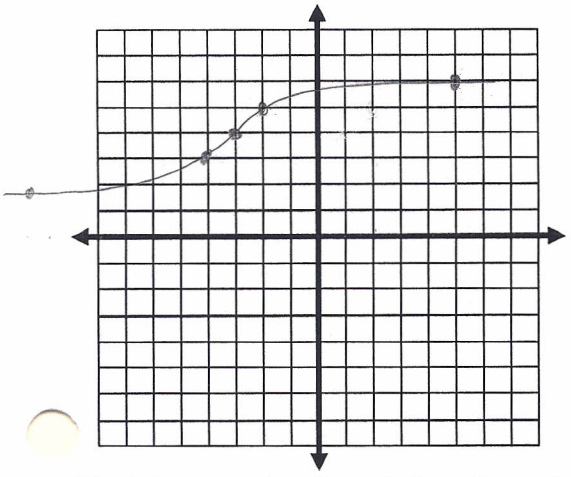
57. Parent graph: $y = \sqrt[3]{x}$

$y = \frac{-1}{5}\sqrt[3]{x-23} + 24$

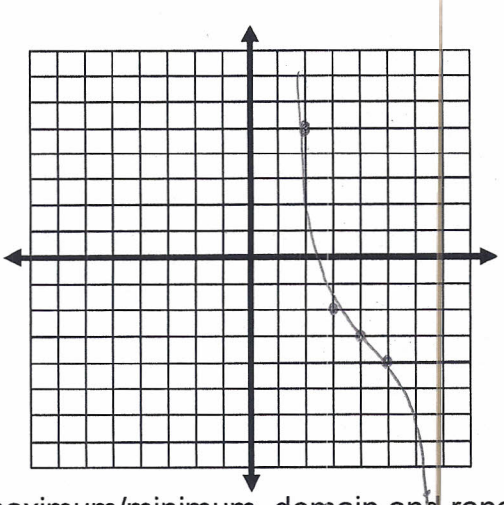
reflect, shrink $\frac{1}{5}$, right 23, up 24

Sketch a graph of each. Describe each transformation.

58. $y = \sqrt[3]{x+3} + 4$ left 3, up 4

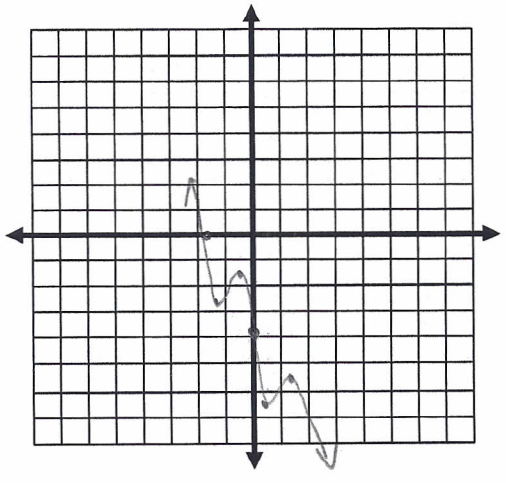


59. $y = -(x-4)^3 - 3$ reflect, right 4, down 3



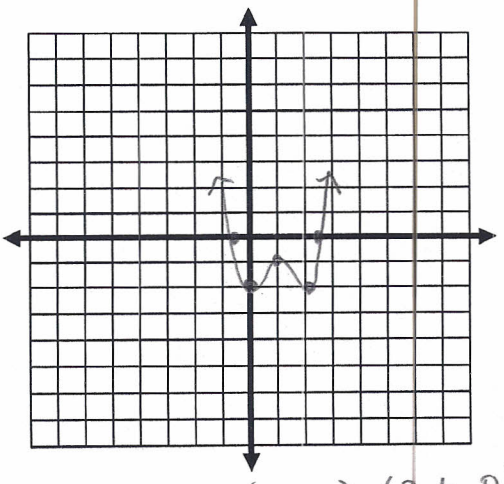
Sketch a graph of each function. Identify the relative maximum/minimum, domain and range, zeros and y-intercept.

60. $f(x) = -x^5 + 4x^3 - 5x - 4$



Zero: (-1.77, 0)
y-int: (0, -4)
min: (-1.37, -2.61), (.73, -6.3)
max: (-.73, -1.7), (1.37, -5.39)
d: $\mathbb{R}$
r: $\mathbb{R}$

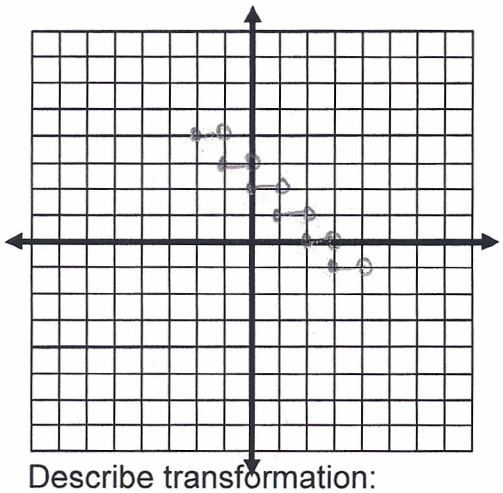
61. $f(x) = x^4 - 4x^3 + 4x^2 - 2$



Zero: (-.6, 0), (2.6, 0)
y-int: (0, -2)
min: (0, -2), (2, -2)
max: (1, -1),
d: $\mathbb{R}$
r: $y \geq -2$

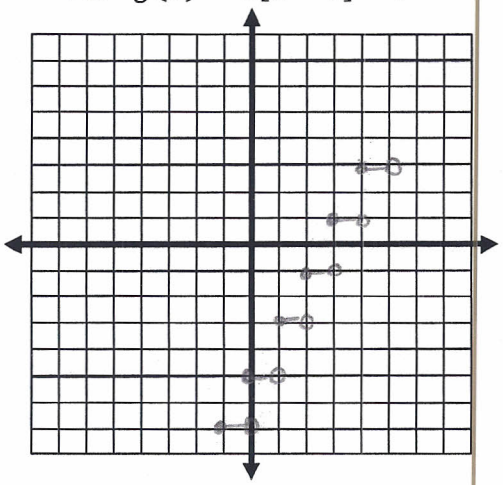
Graph each of the following.

62. $f(x) = -\lfloor x \rfloor + 2$



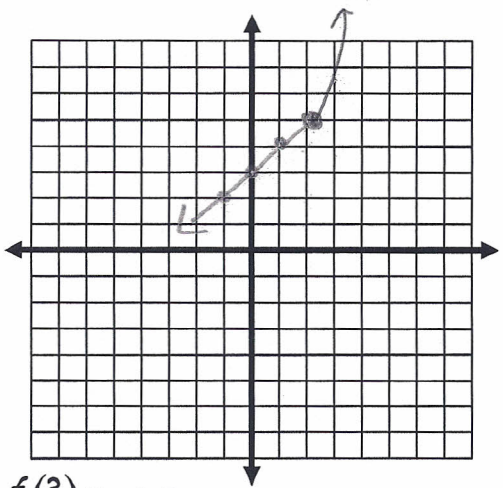
Describe transformation:

63. $g(x) = 2\lfloor x - 1 \rfloor - 3$



Describe transformation:

64. $f(x) = \begin{cases} x + 3, & x < 2 \\ x^2 + 2x - 3, & x \geq +2 \end{cases}$

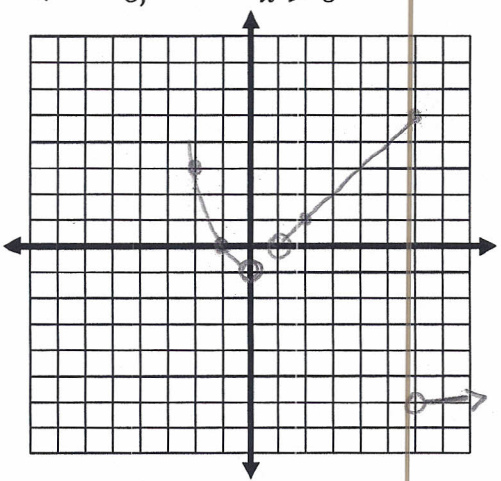


$f(3) =$ 12

$f(-4) =$ −1

$f(-2) =$ 1

65. $f(x) = \begin{cases} x^2 - 1, & x < 0 \\ x - 1, & 1 < x \leq 6 \\ -6, & x > 6 \end{cases}$

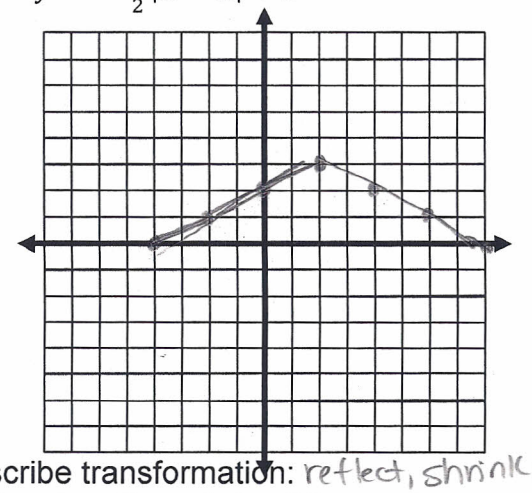


$f(-2) =$ 3

$f(0) =$ ∅

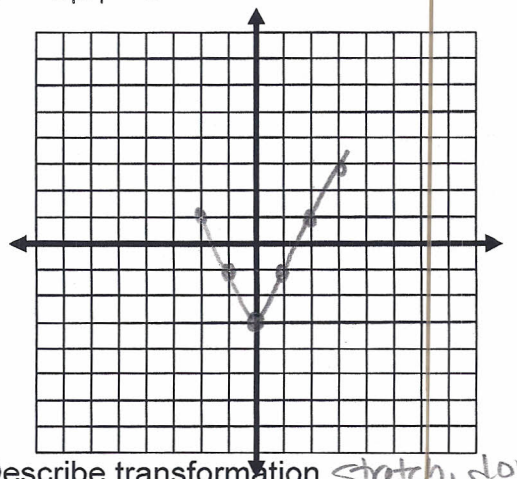
$f(5) =$ 4

66. $y = -\frac{1}{2}|x-2|+3$



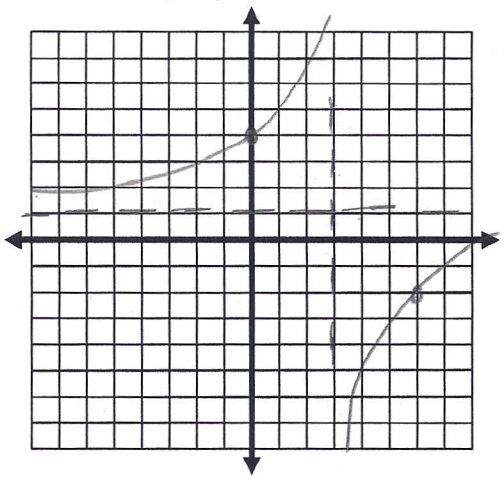
Describe transformation: reflect, shrink, right 2 up 3
Vertex (2,3) Axis of Symmetry: x = 2
Zeros (-4,0), (8,0)

67. $y = 2|x|-3$

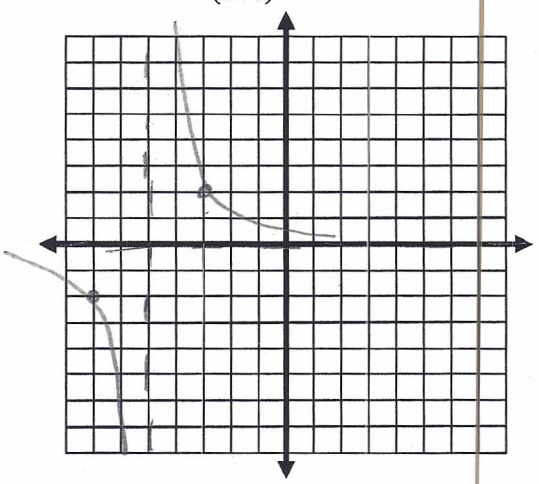


Describe transformation stretch, down 3
Vertex (0,-3) Axis of Symmetry: x = 0
Zeros (1.5,0), (-1.5,0)

68. $f(x) = \frac{-9}{(x-3)}+1$ (sketch asymptotes)



69. $f(x) = \frac{4}{(x+5)}$ (sketch asymptotes)



70. Suppose that x and y vary inversely and that y = ¼ when x = 5. Write a function that models the inverse variation and find y when x = 12. $y = \frac{K}{x}$ $\frac{1}{4} = \frac{K}{5}$ $K = \frac{5}{4}$ $y = \frac{5/4}{12}$ $y = \frac{5}{48}$

71. A drama club is planning a bus trip to New York City to see a Broadway play. The cost per person for the bus rental varies inversely as the number of people going on the trip. It will cost $120 per person if 44 people go on the trip. How much will it cost per person if 60 people go on the trip? Round your answer to the nearest cent, if necessary. $C = \frac{K}{n}$ $120 = \frac{K}{44}$ $K = 5280$ $C = \frac{5280}{60}$ $C = 88$

72. A company has found that the monthly demand for one of its products varies inversely with the price of its product. When the price is $12.50, the demand is 5000 units. Find the demand if the price is reduced to $11.25. $d = \frac{K}{P}$ $5000 = \frac{K}{12.50}$ $K = 62500$ $d = \frac{62500}{11.25}$ $d = 5555.56$

Rewrite in logarithmic form.

73. $3^3 = 27$ $\log_3 27 = 3$

74. $3^{-1} = \frac{1}{3}$ $\log_3 \frac{1}{3} = -1$

75. $7^0 = 1$ $\log_7 1 = 0$

Rewrite in exponential form.

76. $\log 1000 = 3$ $10^3 = 1000$

77. $\log_8 \frac{1}{512} = -3$ $8^{-3} = \frac{1}{512}$

78. $\log_{27} \frac{1}{9} = -\frac{2}{3}$ $27^{-2/3} = \frac{1}{9}$

Evaluate with a calculator. Round to the nearest ten-thousandth.

79. $\log_8 12$ $8^x = 12$ $x = 1.2$

80. $\log_7 7$ $x = 1$ $7^x = 7^1$

81. $\log_6 12$ $6^x = 12$ $x = 1.1799$

Evaluate without a calculator. SHOW YOUR WORK.

82. $\log_2 \frac{1}{8}$ $2^x = 2^{-3}$ $x = -3$

83. $\log_5 \frac{1}{125}$ $5^x = 5^{-3}$ $x = -3$

84. $\log_7 343$ $7^x = 7^3$ $x = 3$

Solve the exponential equation. Round to the nearest ten-thousandth.

85. $14^{3n} = 54$ $n = .5038$

86. $16^n - 2 = 22$ $16^n = 24$ $n = 1.1462$

87. $3 \cdot 12^{x-9} = 36$ $12^{x-9} = 12$ $x - 9 = 1$ $x = 10$

88. If $f(x) = 3^x - 2$ and $g(x) = f(x) + 4$, describe the transformation needed to move from *f(x)* to *g(x)* and write the new equation for *g(x)*. up 4 $f(x) = 3^x + 2$

89. If $f(x) = 6 \cdot \left(\frac{1}{3}\right)^x - 10$ and $g(x) = \frac{1}{2}f(x) - 3$, describe the transformation needed to move from *f(x)* to *g(x)* and write the new equation for *g(x)*. $g(x) = 3\left(\frac{1}{3}\right)^x - 13$

90. Jamaica's population of 2,500,000 in 1988 was expected to grow exponentially by 1.3% each year for the next twenty years. What was in the population in 1993? What was the population in 2008?

$y = 2{,}500{,}000(1 + .013)^x$ 1993 → $x = 5$ 2606780.3

2008 → $x = 20$ 3236897.3

91. A computer depreciates at an average rate of 3.5% per month. If the values of the computer system was originally $12,800, in how many months is it worth $6350?

$6350 = 12{,}800\left(1 - \frac{.035}{12}\right)^{12t}$ $t = 19.999$

92. Cobalt-60 is a radioactive element with a half life of 5.3 years. If you have a 70 gram sample of Cobalt-60 in 2013, when will only 1 gram of the sample remain?

$1 = 70\left(\frac{1}{2}\right)^{t/5.3}$ $t = 32.485$

93. How much would be in an account after 8 years that had a principle balance of $15,000 with a 7.5% annual interest compounded monthly?

$A = 15000\left(1 + \frac{.075}{12}\right)^{12(8)} = \$27280.80$

94. A stock with initial cost of $25 per share triples its price per share per year. What is the price per share after 14 years?

$y = 25(3)^{14} = 119574225$

95. Farmington High is planning its academic festival. All math classes will send 2 representatives to compete in the math bowl. How many different groups of students can be chosen from a class of 16 students?

${}_{16}C_2 = 120$

96. An airline is hiring 5 flight attendants. If 8 people apply for the job, how many different groups of 5 attendants can the airline hire?

${}_8C_5 = 56$

A bag contains 1 green, 4 red, and 5 yellow balls. Two balls are selected at random. Find the probability of each selection.

97. P(2 red) 98. P(1 red and 1 yellow) $\frac{4}{10}\cdot\frac{5}{10}=\frac{20}{100}=\frac{1}{5}$ 99. P(1 green and 1 yellow) $\frac{1}{10}\cdot\frac{5}{10}=\frac{5}{100}=\frac{1}{20}$

100. P(2 green) 101. P(2 red and 1 yellow) **102.** P(1 red and 1 green)

There are 3 nickels, 2 dimes, and 5 quarters in a purse. Three coins are selected in succession at random. Find the probability.

103. P(nickel, then dime, then quarter), if no replacement occurs $\frac{3}{10}\cdot\frac{2}{9}\cdot\frac{5}{8}=\frac{30}{720}=\frac{3}{72}=\frac{1}{24}$

104. P(nickel, then dime, then quarter), if replacement occurs $\frac{3}{10}\cdot\frac{2}{10}\cdot\frac{5}{10}=\frac{30}{1000}=\frac{3}{100}$

105. P(2 nickels, then 1 quarter), if no replacement occurs

106. P(3 dimes), if replacement occurs

107. P(3 dimes), if no replacement occurs

An urn contains 7 white marbles and 5 blue marbles. Four marbles are selected without replacement. Find each probability.

108. P(4 white or 4 blue) $\frac{7}{12}+\frac{5}{12}=1$ 109. P(exactly 3 white) **110.** P(at least 3 white)

111. P(fewer than 3 white) **112.** P(3 white or 3 blue) 113. P(no white or no blue)

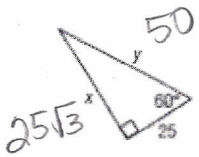
114. A department store employs 28 high school students, all juniors and seniors. Six of the 12 seniors are females and 12 of the juniors are males. One student employee is chosen at random. What is the probability of selecting a senior or a female?

	M	F	
S	6	6	12
J	12	4	16
	18	10	28

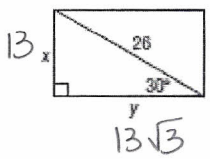
$\frac{12}{28}+\frac{10}{28}-\frac{6}{28}=\frac{4}{7}$

Find each variable. Leave all answers in simplest radical form.

115.

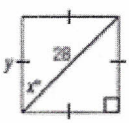


116.



$13\sqrt{3}$

117.

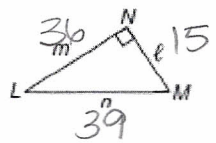


$x=45°$

$y=\frac{28}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{28\sqrt{2}}{2}$

$y=14\sqrt{2}$

118.



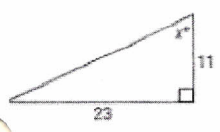
Use L

$\ell=15, m=36, n=39$ find all trig ratios. Leave answer in simplest form.

$\sin L=\frac{15}{39}=\frac{5}{13}$ $\cos L=\frac{36}{39}=\frac{12}{13}$ $\tan L=\frac{15}{36}=\frac{5}{12}$

Find the value of x.

119.



$\tan x=\frac{23}{11}$

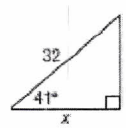
$x=64.4°$

120.

$\sin x=\frac{9}{29}$

$x=18.1°$

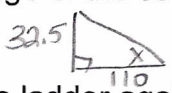
121.



$\cos 41=\frac{x}{32}$

$x=24.2$

122. A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth.



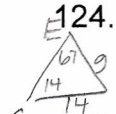
$\tan x = \frac{32.5}{110}$ $x = 16.5°$

123. A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55^0, how far is the ladder from the base of the wall? Round to the nearest foot.

$\tan 55 = \frac{30}{x}$ $x = 21$ ft

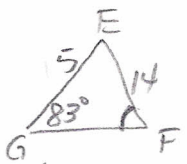
Use the given information to find the missing part of the triangle.

124. $m\angle G = 14^0$, $m\angle E = 67^0$, $e = 14$, find g.



$\frac{g}{\sin 14} = \frac{14}{\sin 67}$ $g = 3.7$

125. $g = 14$, $m\angle G = 83^0$, find $m\angle F$

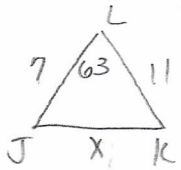


$f = 5$

$\frac{5}{\sin F} = \frac{14}{\sin 83}$

$F = 20.8°$

126. In triangle JKL, $j = 11$, $k = 7$ and $m\angle L = 63^0$, find l.

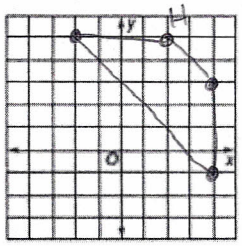


$x^2 = 7^2 + 11^2 - 2(7)(11)\cos 63$

$x = 10.0$

127.

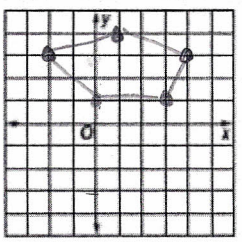
trapezoid $HIJK$ with vertices $H(-2, 5)$, $I(2, 5)$, $J(-4, -1)$, and $K(-4, 3)$ in the y-axis



$H'(2,5)$
$I'(-2,5)$
$J'(4,-1)$
$K'(4,+3)$

128.

pentagon $DEFGH$ with vertices $D(-1, -2)$, $E(2, -1)$, $F(5, -2)$, $G(4, -4)$, $H(1, -4)$ under the translation $(x, y) \rightarrow (x - 1, y + 5)$



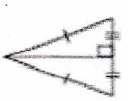
$D'(-2,3)$
$E'(1,4)$
$F'(4,3)$
$G'(3,1)$
$H'(0,1)$

Determine how, if possible, the triangles are congruent.

129.

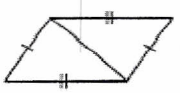
not

130.



HL

131.



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